Microscopic Study of Bubble Growth During Nucleate Boiling

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A photographic technique was used to determine bubble growth rates for ether and pentane boiling on vertical metal surfaces of zinc and an aluminum alloy at atmospheric pressure at saturation conditions. Motion pictures were taken at about 3,000 frames/sec. through a microscope which produced a 13X magnification on the film. The heat flux was varied from about 3,000 to 13,000 B.t.u./(hr.)(sq.ft.). In seven runs out of nine the late-growth equations of Forster and Zuber, Plesset and Zwick, and others were in error by less than 40% for bubble diameters from about 0.1 mm. up to the full field of view of about 0.75 mm. For sizes below 0.1 mm. the growth was too rapid to be measured. Interesting observations which are described quantitatively include bubble vibrations, irregularities in nucleation, and statistical variations in growth rates.

Boiling heat transfer is usually studied by macroscopic observations. Definite progress has resulted from this approach, but the success is far from complete. For nucleate boiling the microscopic viewpoint is attractive. A knowledge of the number of nucleation sites and the rate at which bubbles can grow should permit a prediction of the rate of heat transfer during boiling in a saturated liquid. The identity and the population of active sites were subjects of prior papers (*6, 13*). The present study is concerned with the rate of growth of bubbles at the sites.

The growth of bubbles in boiling liquids has been the subject of considerable theoretical analysis. Attention has been focused both on growth in infinite media and on growth on surfaces. Bosnjakovic (*4*) stated the first growth equation for the case of infinite medium:

$$\frac{dR}{d\theta} = \frac{h\Delta T}{\lambda\rho_v} \qquad (1)$$

The equation is not usable in this form because h is not constant and is not known. Fritz and Ende (*12*) eliminated h by using $h\Delta T = kdT/dx$ and assumed the gradient was identical with that for unsteady state conduction in a slab. This gave Equation (2)

$$R = \frac{2k\Delta T}{\lambda\rho_v\sqrt{\pi\alpha}}\theta^{1/2} \qquad (2)$$

which predicts that radius increases as the square root of time. Plesset and Zwick (22) introduced the equation of motion. Also they assumed the temperature gradient is not that for a slab but rather that for a spherical shell during unsteady state conduction. They obtained Equation (3), for growth in a liquid having uniform initial superheat:

$$R = \sqrt{3}\cdot\frac{2k\Delta T}{\lambda\rho_v(\pi\alpha)^{1/2}}\theta^{1/2} \qquad (3)$$

Forster and Zuber (*10*) used different mathematical techniques and obtained Equation (4):

$$R = \frac{\pi}{2}\cdot\frac{2k\Delta T}{\lambda\rho_v(\pi\alpha)^{1/2}}\theta^{1/2} \qquad (4)$$

Recently the assumption of conduction through an infinite body of expanding liquid having a temperature distribution such that R increases exactly as the square root of time has been used. Scriven (*23*) and also Birkhoff, Margulies, and Horning (3) have used this approach. The equations must be solved by numerical techniques, and no simple general solution can be given. When the liquid superheat is much less than the latent heat of vaporization, and when the liquid density is much greater than the vapor density (both true for the tests reported herein), the general solution simplifies to exactly Equation (3).

None of the above equations should be regarded as correct for bubble growth on a wall. Equations (1) to (4) as well as Scriven's and Birkhoff's analyses were developed for bubble growth in an infinite medium, far away from solid surfaces. Also surface tension, viscosity, and inertia are judged to be unimportant.

Growth on a solid surface is considered by Griffith (*14*), Bankoff and Mikesell (*2*), and Forster (*11*). All of these assume a simple radial flow for the surrounding liquid, which means the bubbles are hemispheres or spheres. In reality the bubbles on a wall rarely take these shapes. Contact angles for bubbles growing in boiling liquids include 30 to 68 deg. for methanol (*20*), 43 deg. for methanol (*24*), 43 to 49 deg. for pentane (*7*), 45 deg. for diethyl ether (*7*), 45 to 60 deg. for Freon-113 (*7*), and 43 deg. for water (*24*).

Each of the theoretical models also includes an assumption concerning the temperature distribution in the liquid. Griffith uses a linear temperature profile from the solid surface out to a particular distance. Bankoff and Mikesell try two profiles: one is linear from the bubble wall out to a particular distance, and one is exponential. Forster uses two profiles, one linear and one exponential, out from the solid wall.

The solutions of Griffith and Bankoff and Mikesell for radius vs. time were found by numerical techniques and cannot be given as simple general equations. Forster gives his results as an integral equation (for linear profile) and an infinite series (for exponential profile). For time near zero the series converges in one term, and Forster's result is identical with Equation (4) except that unity appears as a coefficient in place of $\pi/2$.

Bankoff (*1*) and Forster (*11*) have also treated bubble growth and collapse in a subcooled liquid. These interesting works will not be described here because the present paper is concerned with the case of no subcooling.

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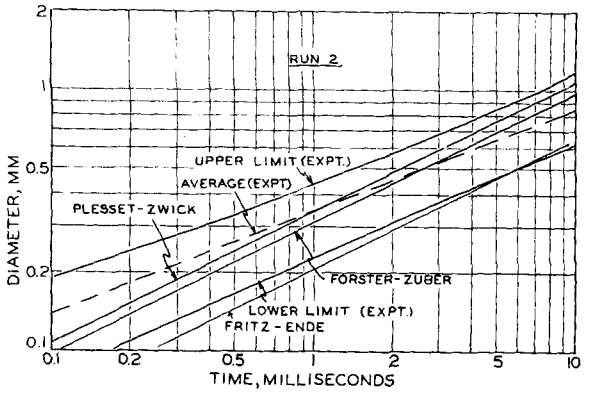

Fig. 1. Envelope of growth curves for eighty-six consecutive bubbles of pentane. Run 2. Three theoretical lines are included for comparison.

Experimental data for bubble growth are scant, in spite of the fact that nearly a doxen persons have made such measurements. Apparently Fritz and Ende in 1936 were the first to make actual quantitative observations. They used photographic techniques, as have all later investigators. Subsequent experimentalists include Gunther and Kreith (*16*), Zmola (*29*), Gunther (*15*), Dergarabedian (*8*), Ellion (*9*), Chun (*5*), McLean, Scherrer, and Faneuff (*21*), Levenspiel (*19*), Harrach (*17*), and Staniszewski (*24*).

The present investigation differs from the preceding in four important aspects. First, the magnification of the image on the present film negatives is greater than for all the prior papers; thus the diameters of small bubbles are known with better accuracy. Second, the identity of the nucleation site is known for each bubble reported herein. Third, two organic liquids were utilized each for the first time. Fourth, the statistical behavior of bubbles was given particular consideration.

EXPERIMENTAL METHOD

The apparatus used has been described previously (*6*). Boiling took place on a flat, vertical, metal surface at atmospheric pressure. The growth of bubbles on the surface was recorded photographically with a microscope perpendicular to the surface and a motion picture camera operating at about 3,000 frames p/sec. The images on the 16-mm. film were thirteen times actual size, in linear dimensions. The films were analyzed frame by frame in a professional editing machine, such that the final images were 100 times actual size. A selection of the motion pictures is available for loan upon request (*28*).

The method for measuring the heat flux and the temperature difference between the surface and the bulk liquid was described earlier (*6*). The liquids used were diethyl ether and normal pentane. There was no liquid subcooling and no artificial agitation. Two metals were used, pure zinc and an aluminum alloy. Bubbles originated at tiny pits and scratches in the metals. These were natural sites and were not introduced deliberately.

For each frame of the motion picture two diameter measurements at right angles to each other were made. Except for the vibrating bubbles the two values agree within 5%. Diameters in this paper are mean values.

ZERO TIME

There is an inherent difficulty in presenting bubble growth data taken from motion pictures. Owing to the discontinuous nature of a framing movie camera the image of the bubble is recorded only at time intervals. Thus the time at which bubble diameter is zero, or zero time, is unknown.

The difficulty of constructing a growth curve, relating the diameter of the growing bubble with time, can be removed by first adopting an arbitrary time scale which is then corrected by shifting it properly. Dergarabedian (*8*) first pointed out that a time shift procedure should be used to determine a separate zero time for each bubble.

An arbitrary time scale may be chosen as the one where time zero is assigned to the last vacant frame before the bubble appears on film. Some authors (*2, 24*) assume true zero time to correspond exactly to this vacant frame, but such a procedure cannot be correct. The action may originate at any instant during the dead time between frames.

The method of selecting the shift depends on the supposed form of the growth equation. For example if the growth curve is $R = a\theta^n$, diameters are plotted against uncorrected time on logarithmic coordinates. Each diameter reading is then shifted along the time axis by a selected constant amount (no greater than the time between two adjoining frames). The shift is selected by trial so that a best straight line results on the graph.

In view of the facts that the existing theoretical expressions for bubble growth on a wall assume idealized bubble models, use unverified temperature distributions, and contain parameters that must be assigned a posteriori, and inasmuch as the goal of this paper is to present new data, a decision was made to present the results in a very simple way. Therefore the best-fit growth equation was assumed to be Equation (5):

$$R = a\theta^n \qquad (5)$$

Equation (5) gives a good fit for the data for all runs. Figure 6 demonstrates the fit for four individual bubbles of Runs 3 and 7. Specific data points are omitted from all other graphs. Complete numerical data (with both uncorrected and corrected times) are available.*

STATISTICAL BEHAVIOR

One reel of film (Run 2) was particularly interesting because it showed eighty-six bubbles of pentane formed consecutively on a zinc surface within a total time of 1.6 sec. A motion picture strip showing the growth of one of these bubbles has been published (*26*). The bubbles were definitely distinct from one another, and there were no collisions. The waiting time between bubbles was enough that the single nucleation site, a 0.031-mm. diameter pit in the zinc surface, could be seen every time. For Run 2 the experimental curves of diameter vs. time for all eighty-six bubbles fall within the boundaries shown in Figure 1. A pronounced variation in growth rates is apparent. With the best-growth equation, Equation (5), used, the value of n varied from 0.312 to 0.512. If R is centimeters and θ is seconds, a varies from 0.139 to 0.625. The statistical nature of the growth parameters is shown in Figure 2. Table 1 shows that a statistical variation occurred for all runs.

The period for a bubble may be expressed as the sum of the growth time on the surface plus the waiting time before the next bubble appears. The periods for the eighty-six bubbles varied significantly. This is shown in Figure 3, wherein the frequency of bubble emission (inverse of the period) extends from 22 to 100 bubbles/sec. The over-all mean is 55.6 with a standard deviation of 13.6.

The waiting time between bubbles is quite variable. Precise values for

* Tabular material has been deposited as document 6887 with the American Documentation Institute, Photoduplication Service, Library of Congress, Washington 25, D. C., and may be obtained for $3.75 for photoprints or $2.00 for 35-mm. microfilm.

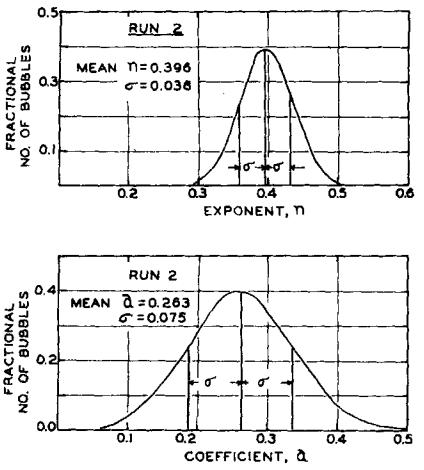

Fig. 2. Variations in growth exponent and growth coefficient for run 2.

the waiting times cannot be determined from the motion pictures, because the view is perpendicular to the heat transfer surface and the instant of bubble break off is therefore obscure. However an estimate is that the waiting time seems to vary by a factor of at least ten to one.

The variability in behavior from one bubble to the next probably is evidence that microscopic eddies exist in the liquid. The superheat of the solid surface was 12°F. for the data in Figure 1. However eddies with less superheat could move across the active site and result in some slow growing bubbles. A fluctuation of 5.2°F. in the superheat of the liquid is sufficient to account for the extremes in the growth rates of Figure 1, if one assumes the dependence on ΔT is as expressed by Plesset and Zwick or Forster and Zuber. Moving eddies should cause optical disturbances because refractive index is sensitive to temperature. Occasional aberrations, apparent wavy motions in the liquid, can be seen in the motion pictures.

An alternate explanation for the variability in bubble growth is that perhaps the temperature of the heat source was fluctuating. Two observations contest this explanation. First, the length of the copper flow path for the conduction of heat from the source to the solid-liquid interface was varied from 2 in. for the present runs to 12 in. for later runs (not recorded in this paper). This change in the damping along the heat flow path caused no detectable differences in bubble growth. Second, three distinct types of heat source at the hot end of the conduction bar have been used: condensing steam, hot flowing oil, and electrical resistance heating. The variable behavior of the bubbles was independent of the type of heat source. It is concluded that the source of bubble growth variability is in the liquid, not in the solid.

COMPARISON WITH THEORY

Two difficulties arise when one tries to compare bubble growth data with theory. One is that real growth varies in a statistical manner, whereas present theories do not account for these variations. Secondly, the expressions which are concerned with growth on walls contain empirical parameters which make difficult direct comparison with data. Equations do exist which contain no empirical constants [Equations (2), (3), (4)], but these are not intended for bubbles growing on walls.

Consequently the Plesset-Zwick equation for bubble growth in the asymptotic period [Equation (3)] will be considered here as a zero-order comparison. The coefficient and exponent in this equation were derived theoretically. The equation is based on numerous assumptions; it is exact only for spherical bubbles, far away from any solid boundary, and surrounded by a large body of superheated liquid. Even if these stringent conditions cannot be met in practice, the equation is useful as an indication of a possible functionality of each variable. Some of the same general dependencies are given by the equations of Forster and Zuber, Bosnjakovic, and others.

Equation (3) predicts that the rate of bubble growth, $dR/d\theta$, is proportional to ΔT. Figure 4 illustrates the actual effect of ΔT for pentane bubbles, and Figure 5 is a similar graph for ether. Each line is the average line (with the mean a and mean n) for a number of bubbles whose statistical nature is indicated in Table 1. Here ΔT is taken to be the difference between the surface temperature of the metal and the saturation temperature of the liquid. The qualitative effect of ΔT is as predicted. Quantitatively however the observed growth coefficient a in Equation (5) is smaller than the coefficient predicted by Equation (3). In a few cases the predicted coefficient is high by an order of magnitude.

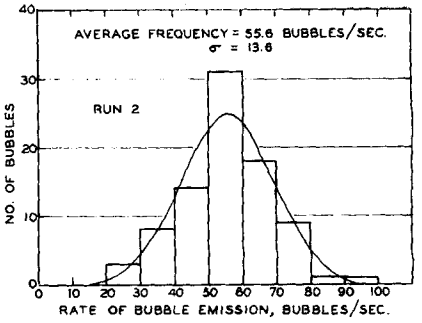

Fig. 3. Variations in bubble emission frequency for eighty-six consecutive bubbles. Run 2.

It is doubtful that the runs at different ΔT are strictly comparable. The six pentane runs in Figure 4 for example were made with six different nucleation sites, identified in Table 1. Disagreement between the two different sets of bubbles (Runs 3 and 4) obtained at the same ΔT, 14°F., suggests that the size of the nucleation site influences bubble growth. The smaller pit produced bigger bubbles than the larger pit at equivalent times. The same conclusion holds for ether. For Run 9 two pits, 0.34 mm. apart, were in the field of view, and bubbles formed sometimes at one and sometimes at the other. Figure 5 shows that the smaller pit produced bigger bubbles. Of course none of the theoretical equations contain any factor dependent on nucleation sites. Future workers must consider that the size (and probably the spacing) of nucleation sites is important when studying the growth of bubbles on solids.

The type of metal used may be important, as suggested by Figure 4. The unexpected intersection of the growth curves for pentane on aluminum with ΔT = 50°F. and pentane on zinc with ΔT = 30°F. is most easily explained by attributing it to the type of metals. No theoretical

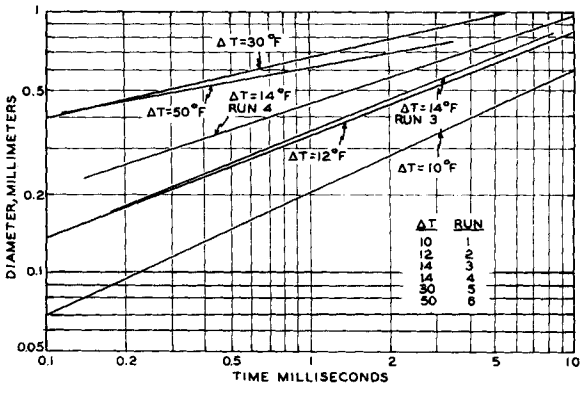

Fig. 4. Effect of ΔT for bubble growth in pentane.

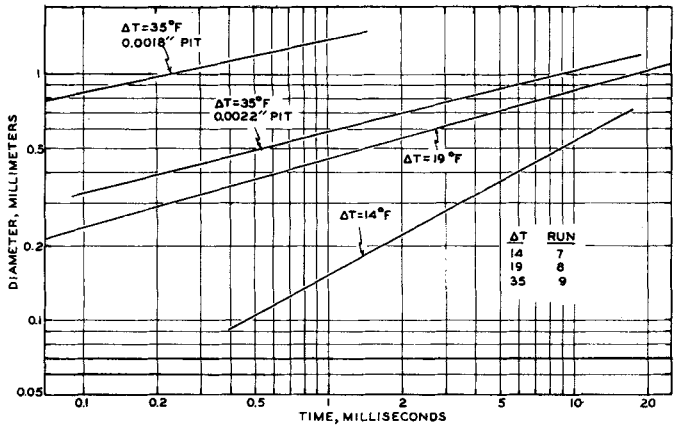

Fig. 5. Effect of ΔT for bubble growth in ether.

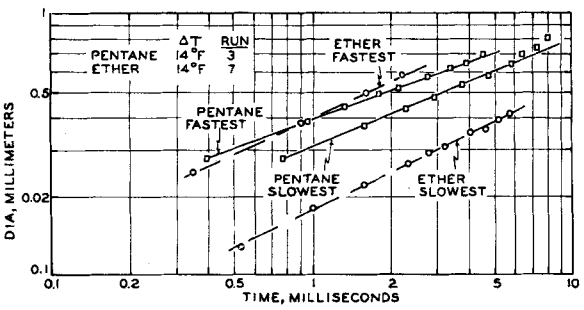

Fig. 6. Comparison of bubble growth for similar liquids at a constant ΔT.

equation exists which predicts this effect.

To prove or disprove the effect of type of metal will be difficult indeed. Data needed will require runs made with identical nucleation sites at the same ΔT on dissimilar metals. No such data exist. The present data cannot be used because the effects of ΔT, size of site, and type of metal are confounded. As future methods are developed for the construction of artificial nucleation sites, it will be possible to test these individual variables.

Equation (3) predicts that the radius of a growing bubble is proportional to the square root of time. Table 1 shows that real bubbles growing on a solid surface tend to grow according to a smaller exponent on time. Only Run 7 had an average exponent as great as 0.5. For the other runs the average n was between 0.19 and 0.48. A distinct decrease of n with increasing ΔT is evident in Table 1 for both test liquids.

Although none of the average values for n and a in Table 1 agree with those predicted by Plesset and Zwick, the Plesset-Zwick equation will give exactly the observed bubble radius at some real time. In other words there exists a simultaneous solution for R and θ in Equations (3) and (5). The last column of Table 1 lists the times at which the values of R agree. In seven runs out of nine this calculated time is about 10^{-4} or 10^{-3} sec. At times within about another order of magnitude of the tabulated times the Plesset-Zwick type of equation gives a rather good fit to the observed data. Consider Run 2 in which the intersection time is 10^{-3} sec. Figure 1 shows the Plesset-Zwick, the Forster-Zuber, and the Fritz-Ende predictions. The first two are satisfactory between 10^{-4} and 10^{-2} sec. in spite of the fact that each has a coefficient and an exponent which disagree with the experimental averages. Even the Fritz-Ende expression is not bad. The intersection times in Table 1 indicate that Runs 1, 6, and 7 are the only ones that may not be represented reasonably by the Plesset-Zwick type of expression. Actually the other six runs (plus Run 1 as well) have bubble diameters which agree within 40% of the Plesset-Zwick prediction at $\theta = 10^{-3}$ sec. At this value of time the prediction for Run 6 is high by 140%, and that for Run 7 is high by 180%. It is concluded that the Plesset-Zwick equation is not a zero-order approximation but is rather a good first approximation for the growth of bubbles on a solid surface in a saturated boiling liquid.

Equation (3) includes five physical properties of the test liquid. It indicates that the growth of bubbles in pentane or in ether should be about the same because these liquids have similar properties. Actually the growth in ether is predicted to be 4% faster than in pentane. Runs were made at 14°F. with both liquids. Figure 6 shows the fastest and slowest growing bubbles for each liquid. This graph depicts the growth of individual, rather than average, bubbles. It demonstrates that the data for any one bubble show a negligible scatter from the best line for that bubble. It demonstrates also that data for the two families of bubbles overlap. When one considers the overlap plus the fact that two different metals were used, no difference between the two liquids can be proved. In the absence of contrary evidence one may assume that the dependence of growth rates on physical properties is correct as shown in Equation (3). Conclusive proof will demand data for liquids of greatly different physical properties boiling on the same nucleation sites on one metal. Such data are not yet obtainable.

BUBBLE HEAT TRANSFER COEFFICIENT

In any discussion of bubble dynamics various factors, such as the

Table 1. Results for Bubble Growth Under Nine Test Conditions

Run no.	Liquid	Metal	ΔT, °F.	Heat flux, $\left(\frac{\text{B.t.u.}}{\text{hr. sq. ft.}}\right)$	Nucleation site, pit diameter, mm.	Bubbles measured	Observed exponent, n Mean	Observed exponent, n Std. dev.	Observed coefficient, a, cm./sec.^{n} Mean	Observed coefficient, a, cm./sec.^{n} Std. dev.	Plesset-Zwick theor., a, cm./sec.$^{1/2}$	Intersection time, sec.
1	Pentane	Zn	10	3,400	0.084	8	0.475	0.059	0.275	0.114	0.45	10^{-9}
2	Pentane	Zn	12	5,400	0.031	86	0.395	0.038	0.265	0.075	0.53	10^{-8}
3	Pentane	Zn	14	4,900	0.059	4	0.405	0.028	0.285	0.071	0.63	3×10^{-4}
4	Pentane	Zn	14	13,400	0.008	2	0.340	0.072	0.255	0.096	0.63	3×10^{-8}
5	Pentane	Zn	30	13,000	0.025	7	0.235	0.095	0.170	0.108	1.37	4×10^{-4}
6	Pentane	Al	50	8,100	0.008	4	0.190	0.026	0.110	0.014	2.36	6×10^{-5}
7	Ether	Al	14	8,900	0.005*	8	0.525	0.075	0.275	0.165	0.65	10^{10}
8	Ether	Zn	19	2,600	0.045	5	0.265	0.067	0.140	0.052	0.90	4×10^{-4}
9	Ether	Zn	35	12,300	0.046	3	0.210	0.062	0.280	0.124	1.69	2×10^{-3}
					0.056	4	0.250	0.053	0.160	0.067	1.69	10^{-4}

* The active site for Run 7 was a scratch.

bubble Reynolds number, Nusselt number, heat transfer coefficient, etc., are of interest. The individual heat transfer coefficient for a growing bubble may be defined by a heat balance, Equation (6):

$$\lambda\rho_v \frac{dV}{d\theta} = hA\Delta T \qquad (6)$$

If the bubble is a sphere, this equation simplifies to

$$h = \frac{\lambda\rho_v}{\Delta T}\frac{dR}{d\theta} \qquad (7)$$

For bubbles obeying the Plesset-Zwick theoretical expression one gets Equation (8):

$$h = \left(\frac{3\rho_L Ck}{\pi\theta}\right)^{1/2} \qquad (8)$$

If one assumes sphericity for real bubbles whose growth is given by $R = a\theta^n$, then Equation (7) becomes Equation (9):

$$h = \frac{\lambda\rho_v na}{\Delta T}\cdot\theta^{n-1} \qquad (9)$$

Any system of consistent units may be used in Equation (9) or the following equations; however a has units which vary from run to run, depending on the value of n.

The value of h for the real bubbles decreases as the bubbles grow. At $\theta = 10^{-4}$ sec. h ranged from 2,400 B.t.u./(hr.) (sq. ft.) (°F.) for Run 6 to 8,400 for Run 4 for the average bubbles of the runs. At 10^{-3} sec. the limiting values were 379 and 1,900. At $\theta = 10^{-2}$ sec. they had decreased to 57 and 420. Obviously the average h at the bubble wall can be improved greatly by removing the bubbles while they are very small. The removal can be achieved either by strong agitation or by the use of great subcooling (in the liquid) to collapse the bubbles before they become large.

As shown, the heat transfer coefficient is a function of time. An average h, for both ideal and real bubbles, can be computed by averaging h with respect to time. It can be shown that for Plesset-Zwick bubbles

$$\bar{h} = \frac{4}{3}h \qquad (10)$$

and for real bubbles

$$\bar{h} = \frac{1}{3}\left(\frac{2n+1}{n}\right)h \qquad (11)$$

The Nusselt number for a bubble is given by $2h\ R/k$. For Plesset-Zwick bubbles obeying Equations (3) and (8)

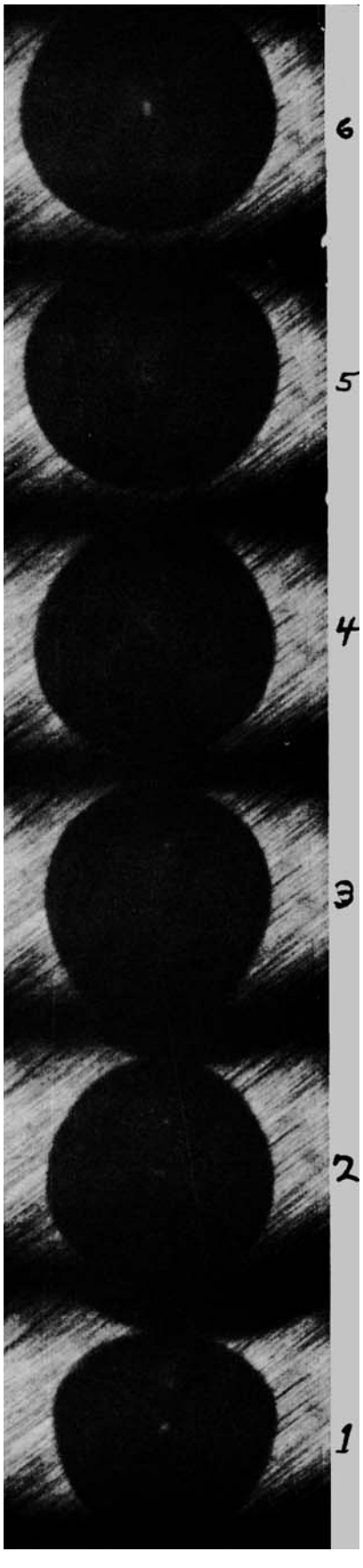

Fig. 7. Microscopic view of pentane bubble vibrating in third mode. The framing rate is 2,520 frames sec. The field of view is 0.5 mm. high.

$$N_{Nu} = \frac{2hR}{k} = \frac{12\rho_L C\Delta T}{\pi\rho_v\lambda} \qquad (12)$$

This Nusselt number is not a function of time or bubble size. For real bubbles the case is different. Expressing h and R as functions of time one obtains the Nusselt number:

$$N = \frac{2hR}{k} = \frac{2\lambda\rho_v na^2}{k\Delta T}\cdot\theta^{(2n-1)} \qquad (13)$$

At first glance Equations (12) and (13) seem to include the physical properties and ΔT in contradictory ways. However a is a function of these parameters, and the disagreement is apparent rather than real. The observed Nusselt numbers ranged from about 2 to 300. For runs with n near 0.5 the Nusselt number was practically constant. For Run 1 $Nu \cong 14$, and for Run 7 $Nu \cong 6$.

The Reynolds number for ideal Plesset-Zwick bubbles is independent of time and diameter. Using Equation (3) one can represent this ideal case by

$$N_{Re} = \frac{2R\rho_L}{\mu}\frac{dR}{d\theta} = \frac{12kc\rho_L{}^2(\Delta T)^2}{\pi\mu\rho_v{}^2\lambda^2} \qquad (14)$$

For the values of ΔT in this study the ideal values of N_{Re} range from 65 to 1,600.

Reynolds numbers for real bubbles are not independent of time (or diameter). Expressing R and $dR/d\theta$ as functions of time one obtains the Reynolds number:

$$N_{Re} = \frac{2\rho_L na^2}{\mu}\cdot\theta^{(2n-1)} \qquad (15)$$

At $\theta = 10^{-4}$ sec. the actual values of N_{Re} ranged from 18 (Run 7) to 2,500 (Run 9, 0.046-mm. pit). At $\theta = 10^{-2}$ sec. the range was 22 to 170.

Is it possible to obtain a good correlation between the Nusselt and Reynolds numbers? Manipulation of the algebra shows that if $R = a\theta^n$, a perfect correlation will result for all values of n. To be more exact the resulting expression, Equation (16), is an identity rather than a correlation. It holds exactly for bubbles obeying the growth expressions of Fritz-Ende, Plesset-Zwick, Forster-Zuber, or any others for which radius is a power function of time:

$$N_{Nu} = N_{Re}\left(\frac{c\mu}{k}\right)\left(\frac{\rho_v\lambda}{c\rho_L\Delta T}\right) \qquad (16)$$

COALESCENCE AND OSCILLATIONS

At a low frequency of bubble emission, collisions between bubbles do not occur. But at higher frequencies collisions become common. A bubble growing at a site expands rapidly

enough to touch the bubble which preceded it from the same site. When this happens with pentane or ether, coalescence results. The new bubble vibrates vigorously as it continues to grow. A film strip depicting coalescence and vibrations during Run 1 has been published (*27*). A strip showing vibrations during Run 3 also has been published (*26*).

Measurements of the rates of oscillation agree reasonably with theoretical predictions as stated by Strasberg (25) and Lamb (*18*). For bubbles having average diameters of roughly 0.25 mm. the second mode vibrations are about 500 to 1,000 cycles/sec. To obtain better accuracy with pentane or ether would necessitate framing rates faster than 4,000 frames/sec.

Most oscillations are of the second mode; however third-mode oscillations were observed in a few cases. Figure 7 illustrates a third-mode vibration for Run 3. In Frames 1, 3, and 5 the bubble is triangular with the point down. In Frames 2, 4, and 6 the point is up. The oscillations seem to be at 1,260 cycles/sec., which is half the framing rate. The third mode theoretical prediction for pentane bubbles of 0.51-mm. diameter (the height of the field of view of Figure 7) gives 1,220 cycles/sec.

Vibration of a bubble also means vibration of the liquid around the bubble. It is wise to question whether heat tranfer through the liquid is improved by bubble vibrations. The motion pictures seem to indicate a faint increase in heat transfer accompanying the vibrations. The effect was too small to be beyond dispute and was too small to be of practical interest.

Although most bubble vibrations were seen to originate from the coalescence of two bubbles, some vibrating bubbles were not accompanied by detectable collisions. The cause of these vibrations is not known.

CONCLUSIONS

1. Bubbles formed at a given nucleation site under a constant superheat show a pronounced variation in growth rates. For growth times between 10^{-4} and 10^{-2} sec. the maximum deviation from the mean rate may amount to $\pm$ 35%.

2. Microscopic bubbles on a surface grow with an exponent on time of less than 0.5, suggesting that the bubbles are not surrounded by a liquid layer of uniform initial superheat.

3. The theoretical equations of Plesset and Zwick and Forster and Zuber for bubbles growing spherically in an infinite medium can be used to predict growth rates of microscopic bubbles at a surface within an error of less than 40%.

4. Bubble emission frequency under constant conditions varies appreciably. This may be due to microscopic eddies in the liquid surrounding the nucleation site.

5. The heat transfer coefficient of real bubbles decreases with time but not at the rate predicted by the Plesset and Zwick equation.

6. Unlike the Plesset and Zwick predictions the Reynolds number and Nusselt number for real bubbles at a given ΔT are not constant but depend on time.

7. The bubbles vibrating owing to collisions and other causes oscillate in a second or third mode. Their oscillation frequencies check well with those predicted by theory.

8. No improvement in growth rates could be detected in oscillating bubbles compared with normal bubbles growing under the same conditions.

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NOTATION

(Dimensions in the $ML\theta TH$ system)

- A = bubble surface area, L^2
- a = empirical coefficient defined by Equation 5, L/θ^n
- c = liquid heat capacity, $H/(MT)$
- h = instantaneous heat transfer coefficient at bubble wall, $H/(\theta L^2 T)$
- $\overline{h}$ = heat transfer coefficient averaged with respect to time, $H/(\theta L^2 T)$
- k = liquid thermal conductivity, $H/(\theta LT)$
- n = empirical exponent defined by Equation (5), dimensionless
- N_{Nu} = bubble Nusselt number, dimensionless
- N_{Re} = bubble Reynolds number, dimensionless
- R = bubble radius, L
- ΔT = liquid superheat at solid-liquid interface, T
- V = bubble volume, L^3
- x = distance, L

Greek Letters

- α = liquid thermal diffusivity, L^2/θ
- λ = latent heat of vaporization, H/M
- μ = liquid viscosity, $M/(\theta L)$
- ρ_l = liquid density, M/L^3
- ρ_v = vapor density, M/L^3
- σ = standard deviation
- θ = time, θ

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